

Going into AP calculus, there are certain skills that have been taught to you over the previous years that we assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

We assume that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example should now be a part of you. If not, you would not be going onto AP calculus. So, only the topics I see that students consistently do not have down in their basic skill set are included here. These are skills that are used continually in A.P. Calculus.

On the following 15 pages, you have 9 to 12 problems per page. Each problem should be done in the space provided. Rather than give you a textbook to remind you of the techniques necessary to solve the problem, I have given you several websites that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, examine these websites. Don't fake your way through these problems. As stated, students are notoriously weak in them, even students who have achieved well prior to AP Calculus. Use the websites.

Realize also that certain concepts are interrelated. Domain, for example, may require you to be expert at working with inequalities. Solving quadratic equations may involve techniques used in solving fractional equations.

This packet is due the first day back in school in the fall. It will be graded. You need to get off to a good start so spend some quality time on this packet this summer. Tear off these first two sheets and return the 15 sheets stapled together. Be sure your name appears on the first sheet. Work needs to be shown when needed. Also do not rely on the calculator. Half of your AP exam next year is taken without the calculator. So paper and pencil techniques only.

It is a mistake to decide to do this now. Let it go until mid-summer. I want these techniques to be relatively fresh in your mind in the fall. Also, do not wait to do them at the very last minute. These take time. If you do two concepts a day, the whole packet will take you about a week to complete.

If you have questions about any of these problems or techniques used in solving them, contact me at the school website/email address. Have a good summer and see you in the fall.

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Topic 1: Fractional & Negative Exponents

Simplify using only positive exponents

1. $-3x^{-3}$

2. $-5\left(\frac{3}{2}\right)(4-9x)^{-1}(-9)$

3. $2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]$

4. $(16x^2y)^{\frac{3}{4}}$

5. $-\frac{x^{-\frac{1}{2}}}{2}\sin\sqrt{x}$

6. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

7. $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$

8. $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$

9. $\left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Write the following absolute value expressions as piecewise expressions

1. $y = |2x - 4|$

2. $y = |6 + 2x| + 1$

3. $y = |4x + 1| + 2x - 3$

Solve the following absolute value inequalities

4. $|x - 3| > 12$

5. $|x - 3| \leq 4$

6. $|10x + 8| > 2$

7. $|3x - 4| > -2$

8. $|x - 6| > -8$

9. $|x + 1| \leq |x - 3|$

Topic 5: Special factorization

Factor completely

1. $x^3 + 8$

2. $x^3 - 8$

3. $27x^3 - 125y^3$

4. $x^4 + 11x^2 - 80$

5. $ac + cd - ab - bd$

6. $2x^2 + 50y^2 - 20xy$

7. $x^2 + 12x + 36 - 9y^2$

8. $x^3 - xy^2 + x^2y - y^3$

9. $(x-3)^2(2x+1)^3 + (x-3)^3(2x+1)^2$

Topic 7: Factor theorem (p over q method/synthetic division)

Use the p over q method and synthetic division to factor the polynomial $P(x)$. Then solve $P(x) = 0$.

1. $P(x) = x^3 + 4x^2 + x - 6$

2. $P(x) = x^3 + 5x^2 - 2x - 24$

3. $P(x) = x^3 - 6x^2 + 3x - 10$

4. $P(x) = x^3 + 2x^2 - 19x - 20$

5. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

6. $P(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$

Topic 9: Solving quadratic equations and quadratic formula

Solve each equation

1. $7x^2 - 3x = 0$

2. $4x(x-2) - 5x(x-1) = 2$

3. $x^2 + 6x + 4 = 0$

4. $2x^2 - 3x + 3 = 0$

5. $2x^2 - (x+2)(x-3) = 12$

6. $x + \frac{1}{x} = \frac{13}{6}$

7. $x^4 - 9x^2 + 8 = 0$

8. $x - 10\sqrt{x} + 9 = 0$

9. $\frac{1}{x^2} - \frac{1}{x} = 6$

Simplify the following

1.
$$\frac{x}{x - \frac{1}{2}}$$

2.
$$\frac{\frac{1}{x} + 4}{\frac{1}{x} - 2}$$

3.
$$\frac{x - \frac{1}{x}}{x + \frac{1}{x}}$$

4.
$$\frac{\frac{3}{x} - \frac{4}{y}}{\frac{4}{x} - \frac{3}{y}}$$

5.
$$\frac{1 - \frac{2}{3x}}{x - \frac{4}{9x}}$$

6.
$$\frac{\frac{x^2 - y^2}{xy}}{\frac{x + y}{y}}$$

7.
$$\frac{x^{-3} - x}{x^{-2} - 1}$$

8.
$$\frac{\frac{x}{1-x} + \frac{1+x}{x}}{\frac{1-x}{x} + \frac{x}{1+x}}$$

9.
$$\frac{\frac{4}{x-5} + \frac{2}{x+2}}{\frac{2x}{x^2 - 3x - 10}} + 3$$

Topic 13: Solving Rational (fractional) equationsSolve each equation for x

1. $\frac{2}{3} - \frac{5}{6} = \frac{1}{x}$

2. $x + \frac{6}{x} = 5$

3. $\frac{x+1}{3} - \frac{x-1}{2} = 1$

4. $\frac{x-5}{x+1} = \frac{3}{5}$

5. $\frac{60}{x} - \frac{60}{x-5} = \frac{2}{x}$

6. $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$

7. $\frac{x}{x-2} + \frac{2x}{4-x^2} = \frac{5}{x+2}$

8. $\frac{x}{2x-6} - \frac{3}{x^2-6x+9} = \frac{x-2}{3x-9}$

9. $\frac{2x+3}{x-1} = \frac{10}{x^2-1} + \frac{2x-3}{x+1}$

Solve each equation on the interval $[0, 2\pi)$

1. $\sin x = \frac{1}{2}$

2. $\cos^2 x = \cos x$

3. $2\cos x + \sqrt{3} = 0$

4. $4\sin^2 x = 1$

5. $2\sin^2 x + \sin x = 1$

6. $\cos^2 x + 2\cos x = 3$

7. $2\sin x \cos x + \sin x = 0$

8. $8\cos^2 x - 2\cos x = 1$

9. $\sin^2 x - \cos^2 x = 0$